ACM ICPC 2016–2017
Northeastern European Regional Contest
Problems Review

December 4, 2016

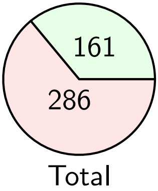
Problems summary

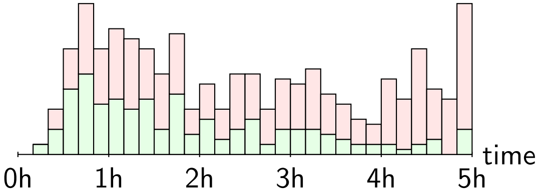
- Recap: 228 teams, 13 problems, 5 hours,
- This review assumes the knowledge of the problem statements (published separately on http://neerc.ifmo.ru/ web site)
- Summary table on the next slide lists problem name and stats
  - **author** — author of the original idea
  - **acc** — number of teams that had solved the problem (gray bar denotes a fraction of the teams that solved the problem)
  - **runs** — number of total attempts
  - **succ** — overall successful attempts rate (percent of accepted submissions to total, also shown as a bar)

Problems summary (2)

problem name	author	acc/runs	succ
Abbreviation	Roman Elizarov	161 /447	36%
Binary Code	Niyaz Nigmatullin	5 /76	6%
Cactus Construction	Pavel Kunyavsky	8 /20	40%
Delight for a Cat	Gennady Korotkevich	1 /2	50%
Expect to Wait	Vitaliy Aksenov	62 /208	29%
Foreign Postcards	Niyaz Nigmatullin	115 /261	44%
Game on Graph	Pavel Kunyavsky	3 /12	25%
Hard Refactoring	Elena Kryuchkova	168 /526	31%
Indiana Jones and the Uniform Cave	Georgiy Korneev	0 /13	0%
Jenga Boom	Georgiy Korneev	59 /396	14%
Kids Designing Kids	Pavel Mavrin	7 /38	18%
List of Primes	Mikhail Dvorkin	12 /28	42%
Mole Tunnels	Borys Minaiev	2 /4	50%

Problem A. Abbreviation





	Java	C++	Python	Total
Accepted	9	142	10	161
Rejected	39	230	17	286
Total	48	372	27	447

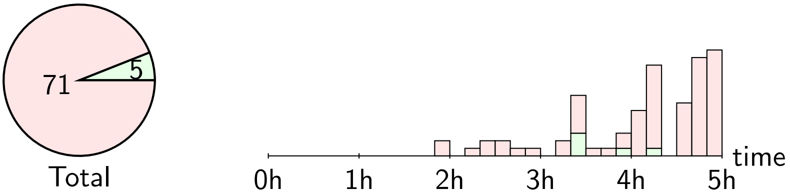
solution	team	att	time	size	lang
Fastest		1	16	2,015	C++
Shortest		1	19	386	Python
Max atts.		9	292	4,689	C++

Problem A. Abbreviation

- One of the two simple problems in the contest
- Many ways to write a solution
- One of the ways is this:
  - Split each line into words and separators
  - Identify *capitalized words* per problem statement
  - Find the longest sequences of two or more words separated by a single space
  - Perform replacements per the problem statement
- The shortest solution from Ural Federal University 4 (Ankudinov, Borzunov, Stikhin) uses a single regular expression with a replacement function:

```
re.sub(r'\b[A-Z][a-z]+( [A-Z][a-z]+\b)+', abbr, text)
```

Problem B. Binary Code



	Java	C++	Python	Total
Accepted	1	4	0	5
Rejected	1	70	0	71
Total	2	74	0	76

solution	team	att	time	size	lang
Fastest		2	204	4,918	Java
Shortest		2	204	4,918	Java
Max atts.		5	205	4,940	C++

Problem B. Binary Code

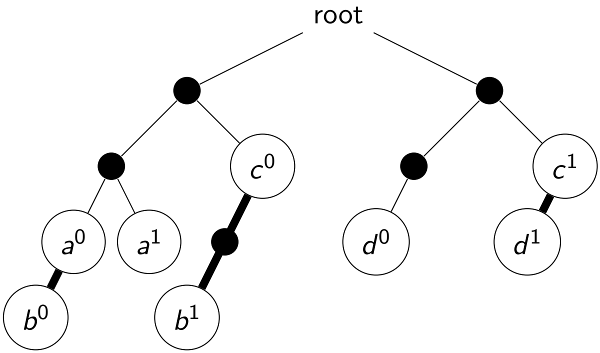
- Solution outline: solve the problem by converting it into an instance of 2-SAT problem
  1. Build a *trie* of the given strings
  2. Define two variables v_i^0 and $v_i^1 = \bar{v}_i^1$ for each word s_i that contains a "?"
     - v_i^0 is *true* and v_i^1 is *false* when "?" is replaced with "0" in s_i
     - v_i^0 is *false* and v_i^1 is *true* when "?" is replaced with "1" in s_i
  3. Create a graph with two nodes for each string. One node for v_i^0, the other for v_i^1
  4. Use the trie to convert binary code constraints into 2-SAT problem instance using *implications*
  5. Use the classical 2-SAT solution algorithm via the graph algorithm to find strongly connected components in *implications graph*

Problem B. Binary Code — Build a trie

- Follow the classic approach, build a *binary* trie
- For strings with "?" add *both* replacements for "?" into a trie
- At the terminal nodes for the string s with "?" put the corresponding variable (s^0 or s^1 depending on replacement)
- At the terminal nodes for the string s *without* "?" put the separate variable T that is always *true*
  - If more than one string without "?" ends at the same node of the trie, the answer is "`NO`"

Problem B. Binary Code — Trie example

- Trie for the first example



a: 00?
b: 0?00
c: ?1
d: 1?0

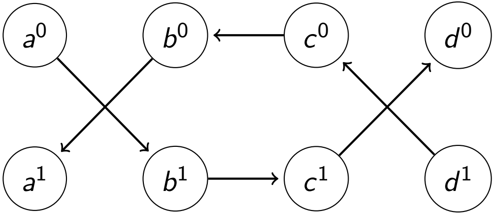
Implications

a^0 **nand** b^0:
- $a^0 \to b^1$
- $b^0 \to a^1$

c^0 **nand** b^1:
- $c^0 \to b^0$
- $b^1 \to c^1$

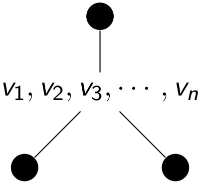
c^1 **nand** d^1:
- $c^1 \to d^0$
- $d^1 \to c^0$

Problem B. Binary Code — Implications graph example



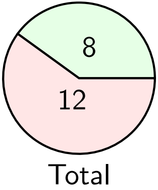
- Classic 2-SAT algorithm finds the answer or decides that it is impossible
- The sample output assigns *true* to a^0, b^1, c^1, d^0

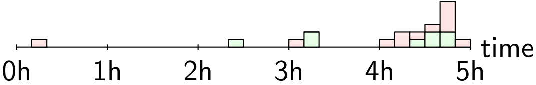
Problem B. Binary Code — Many terminals at node



- Node in a trie can have many terminals (variables) at one node
- At most one of them can be present in a binary code
- We can express this constraint in $O(n)$ implications using n additional variable pairs
  - Define additional variable r_i to be *true* if and only if at least one $v_j, j \geq i$ is *true*
- Or exclude all v_i and v_j pairs, but return "`NO`" answer when n is more than the depth of this node in a trie plus one

Problem C. Cactus Construction



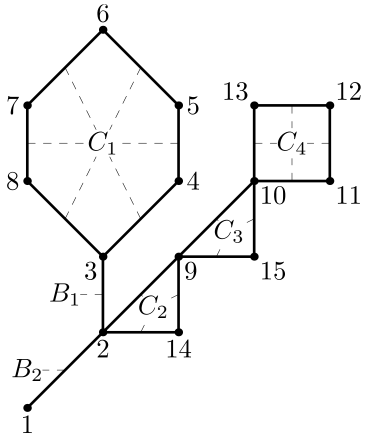


	Java	C++	Python	Total
Accepted	0	8	0	8
Rejected	0	12	0	12
Total	0	20	0	20

solution	team	att	time	size	lang
Fastest		1	142	4,953	C++
Shortest		1	261	2,512	C++
Max atts.		4	275	6,786	C++

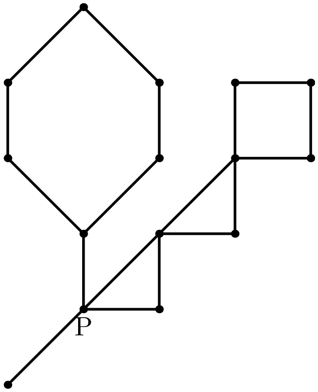
Problem C. Cactus Construction (1)

- Depth-first search (DFS) of the cactus to split the edges of the cactus into disjoint sets of *bridges* B_i and *cycles* C_i
  - Each back edge found during DFS signals a cycle
  - All edges that do not belong to any cycle are bridges



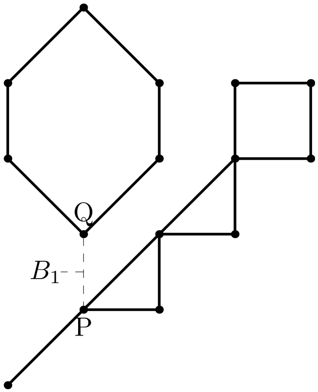
Problem C. Cactus Construction (2)

- Recursive procedure: given a cactus and a vertex P in it, construct the cactus in such a way that all vertices have color 2 except P, which must have color 1.
- Pick any vertex as P to start the procedure.



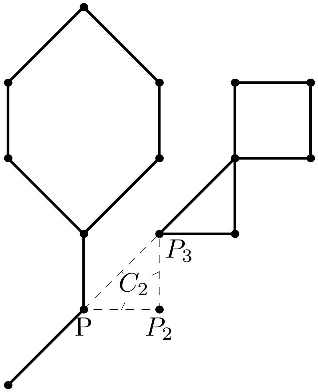
Problem C. Cactus Construction (3)

- If there's a bridge B_i connecting P with some other vertex Q:
  - Remove the bridge B_i.
  - Recursively construct two halves using P and Q as designated vertices (color 1).
  - Build edge B_i and color Q with 2 (more details later).



Problem C. Cactus Construction (4)

- If there's a cycle C_i passing through $P = P_1, P_2, \ldots, P_k$:
  - Remove all edges of C_i.
  - Graph splits into k components. Recursively construct them using P_j as designated vertices (color 1).
  - Build all edges of C_i and color $P_2, P_3, \ldots, P_k$ with 2 (more details later).

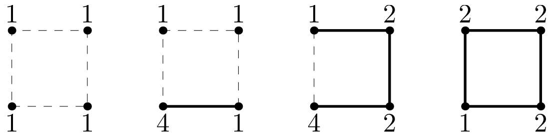


Problem C. Cactus Construction (5)

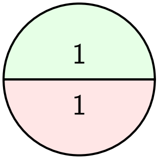
- How to build a new bridge connecting two components with designated vertices P and Q:
  - Initially P and Q are colored with 1, the rest with 2.
  - Recolor 1 to 3 in the component of Q.
  - Join components of P and Q.
  - Connect colors 1 an 3, connecting P and Q.
  - Recolor 3 to 2.

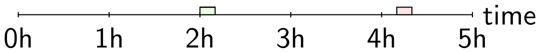
Problem C. Cactus Construction (6)

- How to build a new cycle connecting k components with designated vertices $P = P_1, P_2, \ldots P_k$:
  - Build edges of the cycle one by one, starting with the edge between P_1 and P_2.
  - Each edge except the last one is built as a new bridge.
  - But remember to recolor the first vertex P_1 to 4 instead of 2 after building the first edge.
  - This allows to close the cycle in the end by connecting colors 1 and 4.



Problem D. Delight for a Cat





	Java	C++	Python	Total
Accepted	0	1	0	1
Rejected	0	1	0	1
Total	0	2	0	2

solution	team	att	time	size	lang
Fastest		1	123	2,825	C++
Shortest		1	123	2,825	C++
Max atts.		1	123	2,825	C++

Problem D. Delight for a Cat

- Out of every k consecutive hours, the cat must sleep at least min_s hours and eat at least min_e hours
  - Let $max_e = k - min_s$, now the cat must eat between min_e and max_e hours out of every k
- Let's say that the cat is sleeping by default, and it gets $\delta_i = e_i - s_i$ delight for eating at hour i
  - In the end, add $\sum_{i=1}^{n} s_i$ to the answer

Problem D. Delight for a Cat (2)

- Let $min_e = 0$, $max_e = 1$
- Dynamic programming:
  - Let f_i be the maximum amount of delight for hours from i to n
  - $f_i = max(f_{i+1}, \delta_i + f_{i+k})$
  - Here, we define $f_i = 0$ for $i > n$
- Dynamic programming $\rightarrow$ shortest path:
  - Vertices S, T and $1, 2, \cdots, n$
  - Edge from vertex i to vertex $i + 1$ (or T, if $i + 1 > n$) with cost 0
  - Edge from vertex i to vertex $i + k$ (or T, if $i + k > n$) with cost $-\delta_i$
  - Edges from vertex S to vertices $1, 2, \cdots, k$ with cost 0
  - Negated length of the shortest path from S to T is the answer

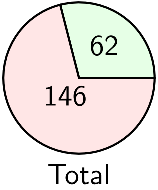
Problem D. Delight for a Cat (3)

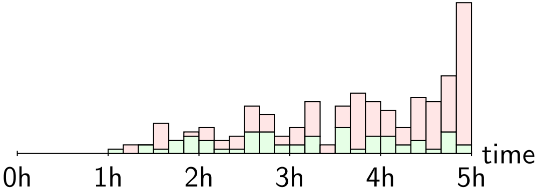
- Let $min_e = 0$, $max_e \geq 0$
- Graph for $max_e = 1 \rightarrow$ network for $max_e \geq 0$:
  - Vertices S, T and $1, 2, \cdots, n$
  - Edge from vertex i to vertex $i + 1$ (or T, if $i + 1 > n$) with cost 0 and capacity max_e
  - Edge from vertex i to vertex $i + k$ (or T, if $i + k > n$) with cost $-\delta_i$ and capacity 1
  - Edges from vertex S to vertices $1, 2, \cdots, k$ with cost 0 and capacity ∞
  - Negated minimum cost of flow of value max_e from S to T is the answer

Problem D. Delight for a Cat (4)

- Let $max_e \geq min_e \geq 0$
- Network for $min_e = 0 \rightarrow$ network for $min_e \geq 0$:
  - Vertices S, T and $1, 2, \cdots, n$
  - Edge from vertex i to vertex $i + 1$ (or T, if $i + 1 > n$) with cost 0 and capacity $max_e - min_e$
  - Edge from vertex i to vertex $i + k$ (or T, if $i + k > n$) with cost $-\delta_i$ and capacity 1
  - Edges from vertex S to vertices $1, 2, \cdots, k$ with cost 0 and capacity ∞
  - Negated minimum cost of flow of value max_e from S to T is the answer

Problem E. Expect to Wait

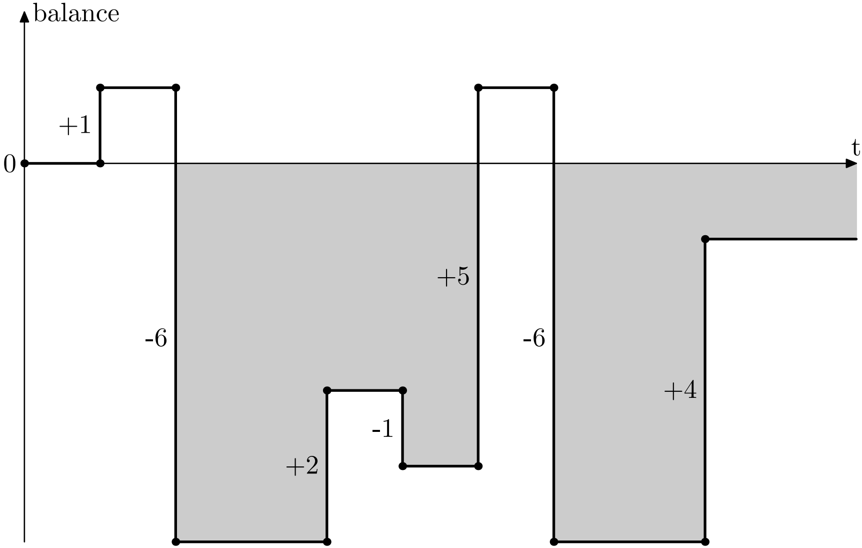




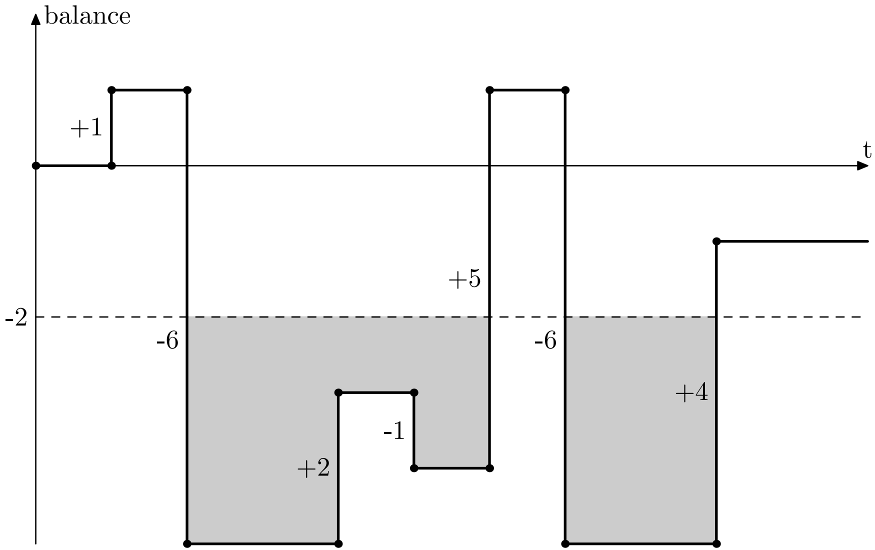
	Java	C++	Python	Total
Accepted	3	59	0	62
Rejected	3	143	0	146
Total	6	202	0	208

solution	team	att	time	size	lang
Fastest		1	61	2,093	C++
Shortest		1	115	1,274	C++
Max atts.		6	285	2,167	C++

Problem E. Expect to Wait



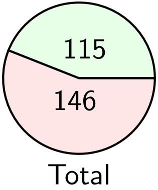
Problem E. Expect to Wait (2)

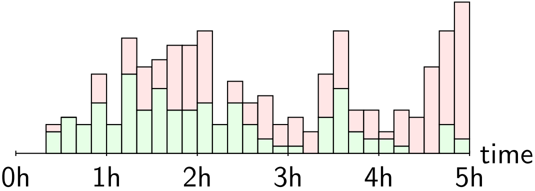


Problem E. Expect to Wait (3)

- For b_i, calculate square under line $balance = -b_i$
- Scan-line
- O(NlogN + QlogQ)

Problem F. Foreign Postcards





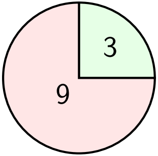
	Java	C++	Python	Total
Accepted	3	112	0	115
Rejected	17	126	3	146
Total	20	238	3	261

solution	team	att	time	size	lang
Fastest		1	23	1,774	C++
Shortest		1	68	483	C++
Max atts.		15	281	1,030	C++

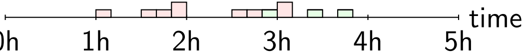
Problem F. Foreign Postcards

- A_i is the expected number of `W`'s starting from position i.
- $A_i = \frac{1}{n-i} \sum_{j=i+1}^{n} A_j + |\{k \mid k \in [i, j) \wedge S_i \neq S_k\}|$
- Straightforward calculation — $O(N^2)$ solution.
- The second sum is $\sum_{j=i+1}^{n} [S_i \neq S_j] \cdot (n - j)$
- It is equal to either $\sum_{S_j = C} (n - j)$ or $\sum_{S_j = W} (n - j)$
- Calculate everything in linear time, $O(N)$ solution.

Problem G. Game on Graph



Total



	Java	C++	Python	Total
Accepted	0	3	0	3
Rejected	0	9	0	9
Total	0	12	0	12

solution	team	att	time	size	lang
Fastest		7	174	3,103	C++
Shortest		2	221	2,023	C++
Max atts.		7	174	3,103	C++

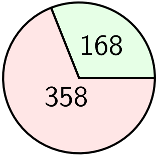
Problem G. Game on Graph

- Let's name a pair of vertex and player who has move now a *position* in game.
- If first player can enforce draw, regardless of second player moves, he will do it.
- If second player can enforce his winning, regardless of first player moves, he will do it.
- If first player can't enforce draw, second player will not allow draw happen, because it's worst result for him
- If second player can't enforce winning, first player will not allow him to win, because it's worst result for him.
- So, in all other positions, first player will win.

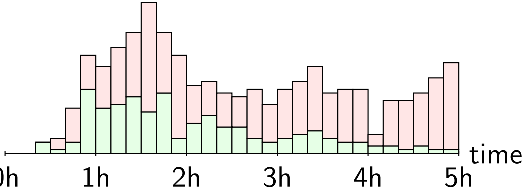
Problem G. Game on Graph (2)

- How to find all positions, where first player can enforce draw?
- Let's put in queue all positions, where player have no moves.
- While queue is not empty, get next position from queue.
- If it's first player turn, than mark all positions of second player with move to this posision and put them to queue.
- If it's second player turn, than mark all positions of first player, where it's last move to non-marked positions, and put them to queue.
- First player can enforce draw, iff position is not marked.
- Postions, where second player can enforce win, can be found in similar way.

Problem H. Hard Refactoring



Total



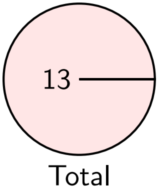
	Java	C++	Python	Total
□ Accepted	10	154	4	168
□ Rejected	29	318	11	358
Total	39	472	15	526

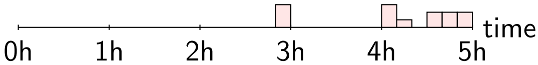
solution	team	att	time	size	lang
Fastest		1	20	1,982	C++
Shortest		2	109	1,303	Python
Max atts.		11	242	2,721	C++

Problem H. Hard Refactoring

- It is an easy problem. The hardest part is parsing
- Once input is parsed, it is Ok to simply fill a Boolean array of 2^{16} items, then print the answer
- The only tricky thing in this problem are the edges of the set of 16-bit integers and the corresponding samples are given in the problem statement

Problem I. Indiana Jones and the Uniform Cave



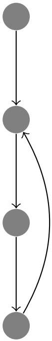


	Java	C++	Python	Total
Accepted	0	0	0	0
Rejected	0	13	0	13
Total	0	13	0	13

Problem I. Indiana Jones and the Uniform Cave

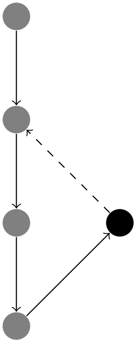
- Solution overview: use depth-first-search (DFS) to recursively traverse the graph
  - Mark with "`right`" chambers on the current DFS path to root. Let us call them *gray* chambers
  - Mark with "`left`" chambers that were already visited. Let us call them *black* chambers
- At each node do "`1 right 1`" m times to check all outgoing passages
- Keep the track of the highest gray ("`right`") chamber encountered down from the current chamber in dfs and the number of the corresponding passage in the chamber
- When all m passages out of the chamber are visited, follow to that chamber until gray ("`right`") chamber is encountered and follow down to the previous chamber

Problem I. Indiana Jones and the Uniform Cave (2)



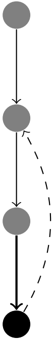
- When gray (“`right`”) chamber is encountered:
- Mark it with “`left`”
- Follow gray (“`right`”) chambers with “`0 right 0`” until the chamber previously marked with “`left`” is reached
- Count the number of passages visited
- Make another pass, taking one fewer passage to “backtrack” and put stones at “`right`” again
- Remember how many gray chambers up the path we’ve got to!

Problem I. Indiana Jones and the Uniform Cave (3)



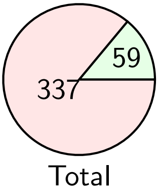
- When black (“`left`”) chamber is encountered:
- Follow black (“`left`”) chambers with “`0 left 0`”
- Follow gray (“`right`”) chambers with “`0 right 0`”
- Count the number of passages visited
- Make another pass, taking one fewer passage to “backtrack”
- Put stones at “`right`” or “`left`” as they were
- Remember how many gray chambers up the path we’ve got to!

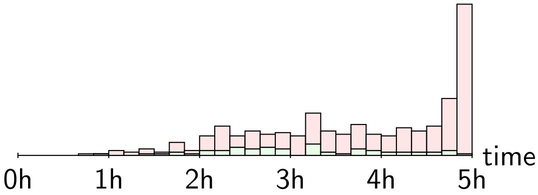
Problem I. Indiana Jones and the Uniform Cave (4)



- When leaving the chamber in dfs (after visiting all *m* passages), use the passage that is leading to the highest gray chamber
- Stones were properly left in place previously, just follow them
- Mark the chamber we are leaving as black (“`left`”)

Problem J. Jenga Boom





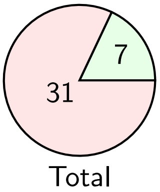
	Java	C++	Python	Total
Accepted	2	57	0	59
Rejected	21	316	0	337
Total	23	373	0	396

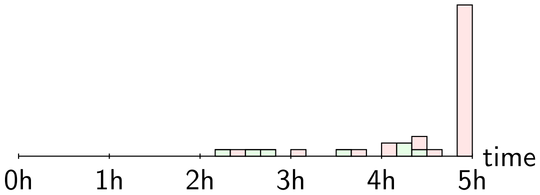
solution	team	att	time	size	lang
Fastest		1	55	1,919	C++
Shortest		1	138	1,707	C++
Max atts.		14	198	2,669	C++

Problem J. Jenga Boom

- The solution is straightforward
- Keep the sum of coordinates of centers of remaining blocks at each level
- Keep the set of remaining blocks (in a Boolean array)
- When a block is removed, update this information, then recheck stability condition for every level (top to bottom)
  - Sum centers of masses and count total number of blocks above the current cross-section
  - Use the set of remaining block at the level immediately below the current corss-section to compute its convex hull in a trivial way
- Some tips
  - The number w is irrelevant to the problem
  - It is easier to compute everything in 64-bit integer numbers
  - Do not do binary search on answer! Simulate every block removal

Problem K. Kids Designing Kids





	Java	C++	Python	Total
Accepted	0	7	0	7
Rejected	3	28	0	31
Total	3	35	0	38

solution	team	att	time	size	lang
Fastest		1	133	3,236	C++
Shortest		1	258	2,613	C++
Max atts.		4	252	3,682	C++

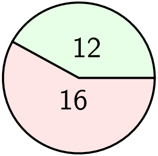
Problem K. Kids Designing Kids

- Find the top-left freckle in each of three given pictures.
- We'll prove that after moving the figures, some two of these three freckles must be in the same point.
- There are only three possible shifts, check them all.
- To check if two pictures are the same, again find top-left freckles in each of them. These freckles must be in the same point.

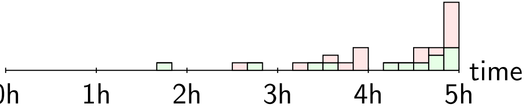
Problem K. Kids Designing Kids — Proof

- Problem is equal to the following: move figures A, B and C in such a way that $A \oplus B \oplus C = \emptyset$ (empty figure).
- Let's look on top-left freckle in each picture. Suppose that after moving, they are in different points.
- Now let's find the top-left freckle from these three.
- This freckle will be present in the final symmetrical difference $A \oplus B \oplus C$, because it cannot be denied by any other freckle.

Problem L. List of Primes



Total



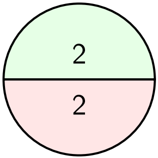
	Java	C++	Python	Total
Accepted	0	12	0	12
Rejected	0	16	0	16
Total	0	28	0	28

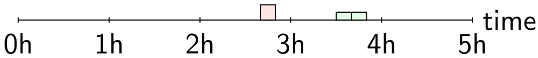
solution	team	att	time	size	lang
Fastest		1	108	3,093	C++
Shortest		1	108	3,093	C++
Max atts.		4	294	4,471	C++

Problem L. List of Primes

- Recursive procedure — output all sets with the following properties:
  - from the first x primes the set *prefix* is selected;
  - all the other selected primes add up to *sum*.
- $output(x, prefix, sum)$
  - $output(x + 1, prefix + [p_{x+1}], sum - p_{x+1})$
  - $output(x + 1, prefix, sum)$
- If the output is to left from the desired segment, skip it without going deeper in recursion.
- Precalculte number and total length of all sets in which:
  - the first x primes are not used;
  - selected primes add up to *sum*.
- Run $output(0, [], sum)$ for $sum = 2, 3, 4, \ldots$

Problem M. Mole Tunnels



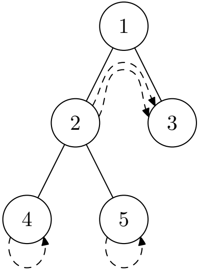


	Java	C++	Python	Total
Accepted	0	2	0	2
Rejected	0	2	0	2
Total	0	4	0	4

solution	team	att	time	size	lang
Fastest		3	215	3,397	C++
Shortest		1	227	2,185	C++
Max atts.		3	215	3,397	C++

Problem M. Mole Tunnels

- Cannot be solved with Minimum Cost Maximum Flow algorithm (too slow)
- We need to find shortest path in a faster way
- Calculate dynamic programming: shortest path in a subtree of vertex v
- Iterate over all possible LCA to find next shortest path
- After sending flow along augmenting path dynamic programming values changes only for at most 40 vertices



Credits

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